LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc., DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2013**

MT 1500 – ALGEBRA, ANALY.GEO., CALCULUS & TRIGONOMETRY

Date : 12/11/2013 Dept. No. Max. : 100 Marks
Time : 1:00 - 4:00

PART - A

ANSWER ALL THE QUESTIONS: $(10\ x\ 2 = 20\, marks)$

1. Find the n^{th} differential coefficient of $(ax+b)^m$.
2. Find the slope of the tangent with the initial line for the cardioid $r = a(1-\cos\theta)$ at $\theta = \pi/6$.
3. Write the cartesian formula for the radius of curvature.
4. Find the p-r equation of the curve $r = a\sin\theta$.
5. Find the equation with rational coefficients whose roots are 1, $\left(3-\sqrt{-2}\right)$.
6. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$,find the value of $\sum \alpha^2$.
7. Write down the expansion of $\sin 5\theta$.
8. Prove that $\cosh^2 x - \sinh^2 x = 1$.
9. Define conjugate diameter of an ellipse.
10. Find asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.

PART – B

ANSWER ANY FIVE QUESTIONS: **(5 x 8 = 40 marks)**

11. Find the n^{th} differential coefficient of $\sin^7\theta\cos^5\theta$.
12. Find the angle of intersection of the cardioids $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$.
13. Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$ and $y = a(1-\cos\theta)$ is $4a\cos\dfrac{\theta}{2}$.
14. Solve the equation $81x^3 - 18x^2 - 36x + 8 = 0$ whose roots are in harmonic progression.
15. Find the roots of the equation $x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0$.
16. If $\sin(A+iB) = x+iy$, prove that i) $\dfrac{x^2}{\cosh^2 B} + \dfrac{y^2}{\sinh^2 B} = 1$ and ii) $\dfrac{x^2}{\sin^2 A} - \dfrac{y^2}{\cos^2 A} = 1$.
17. If P and D are extremities of conjugate diameters of the ellipse, show that the locus of the point of intersection of the tangents at P and D is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 2$.
18. Derive the polar equation $\dfrac{l}{r} = 1 + e\cos\theta$ of a conic.

PART – C

ANSWER ANY TWO QUESTIONS: **(2 x 20 = 40 marks)**

19. a) If $y = \sin(m\sin^{-1} x)$,prove that $(1-x^2)y_2 - xy_1 + m^2 y = 0$ and
$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

b) Show that, in the parabola $y^2 = 4ax$,the subtangent at any point is double the abscissa and the subnormal is constant.

20. a) Show that the radius of curvature at any point on the equi angular spiral $r = ae^{\theta\cot\alpha}$ is $r\,\mathrm{cosec}\,\alpha$.

b) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$. Find the value of $(\alpha^2+1)(\beta^2+1)(\gamma^2+1)$.

21. a) Find the real roots of $x^3 - 3x + 1 = 0$ to three places of decimal using Horner's rule. b)Prove that $\dfrac{\sin 7\theta}{\sin\theta} = 64\cos^6\theta - 80\cos^4\theta + 24\cos^2\theta - 1$.

22. a)Sum to infinity $c\sin\alpha - \dfrac{c^2}{2}\sin 2\alpha + \dfrac{c^3}{3}\sin 3\alpha + \ldots\infty$.

b) If e, e_1 are the eccentricities of a hyperbola and its conjugate, show that $\dfrac{1}{e^2} + \dfrac{1}{e_1^2} = 1$.

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